Debt, Taxes, and Liquidity∗

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Abstract

We analyze a dynamic model of optimal capital structure and liquidity management when firms face external financing frictions. Besides the classical tradeoff between the tax advantages of debt and bankruptcy costs, an important new cost of debt financing in this context is an endogenous debt servicing cost: debt payments drain the firm’s valuable liquidity reserves and thus impose higher expected external financing costs on the firm. The precautionary demand for liquidity also means that realized earnings are separated in time from payouts to shareholders, implying that the classical Miller formula for the net tax benefits of debt no longer holds. Our model offers a novel perspective for the “debt conservatism puzzle” by showing that financially constrained firms choose to limit debt usages in order to preserve their liquidity. In some cases, they may not even exhaust their risk-free debt capacity.

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1 Introduction

We develop a dynamic tradeoff theory of capital structure for financially constrained firms, by evaluating the classical tax versus bankruptcy cost balance in a dynamic setup where the firm faces external financing costs. The fundamental change from the classical dynamic tradeoff theory that follows from the introduction of external financing costs is that the firm optimally retains some of its earnings and faces a precautionary liquidity management problem.¹ That is, the financially constrained firm faces both an asset and a liability management problem.

When the financially constrained firm issues debt it incurs a debt servicing cost – the cash drain associated with debt services including interest payments. This cost is over and above the expected bankruptcy cost associated with debt that are traditionally emphasized. As a result, we show that the financially constrained firm optimally issues less debt than an unconstrained firms with no external financing costs. We thus provide a novel perspective on the “debt conservatism puzzle” documented in the empirical capital structure literature (see Graham (2000, 2003)).

The firm’s precautionary savings motive, by separating the times when earnings are realized and when they are paid out, introduces another dimension to the standard tradeoff theory. By retaining its earnings, the firm is making a choice on behalf of its shareholders to defer the payment of their personal income tax liabilities on this income. As Harris and Kemsley (1999), Collins and Kemsley (2000), Lewellen and Lewellen (2006), and Frank, Singh, and Wang (2010) have pointed out, when firms choose to build up corporate savings, personal taxes on future expected payouts must be capitalized, and this tax capitalization changes both the market value of equity and the net tax benefit calculation for debt.

Specifically, according to the ubiquitous Miller formula, the marginal net tax benefit of debt is measured by

\[
\text{Marginal net tax benefit of debt} = \frac{(1 - \tau_i) - (1 - \tau_e) \times (1 - \tau_c)}{(1 - \tau_i)},
\]

¹Corporate cash holdings of U.S. publicly traded non-financial corporations are known to represent a substantial fraction of corporate assets, as Bates, Kahle, and Stulz (2009) have documented.
where $\tau_c$, $\tau_i$, and $\tau_e$ denote the tax rate for corporate income, personal interest income, and personal equity income, respectively. We show that the marginal net tax benefit of debt for a financially constrained firm is:

\[
\text{Marginal net tax benefit of debt} = \frac{(1 - \tau_i) - \text{Marginal equity value of liquidity} \times (1 - \tau_c)}{(1 - \tau_i)}.
\]

In the presence of financial constraints, the marginal equity value of liquidity is equal to $(1 - \tau_e)$ only when the firm chooses to pay out and otherwise takes higher values. Thus, the Miller formula only holds when realized earnings are immediately paid out, that is, when the firm is unconstrained. When the firm prefers to retain its earnings, the net tax benefit of debt are generally lower than the one implied by the Miller formula. We also show that the marginal net tax benefit of debt can become substantially negative when the firm has low corporate savings and is at risk of running out of cash. This is not just a conceptual observation; it is quantitatively important as the firm is almost always seeking to augment its liquidity holding and is rarely sufficiently flush with cash to be able to afford immediately paying out all its realized earnings.

It may actually be tax-efficient sometimes to let shareholders accumulate savings inside the firm, as Miller and Scholes (1978) have observed. Thus, the tax code influences not just leverage policies but also corporate savings, which is captured by our model. Moreover, the optimal cash management policies in our model have important implications for standard corporate valuation methods such as the adjusted present value method (APV, see Myers (1974)), which are built on the assumption that the firm does not face any financial constraints. The APV method is commonly used to value highly levered transactions, as for example in the case of leveraged buyouts (LBOs). A standard assumption when valuing such transactions is that the firm pays down its debt as fast as possible (that is, it does not engage in any precautionary savings). Moreover, the shadow cost of draining the firm of cash in this way is assumed to be zero. As a result, highly levered transactions tend to be overvalued and the risks that the firm may run into liquidity problems (e.g., being forced to raise costly external funds or liquidating assets through fire sales) are not adequately accounted for by this method.
The model we consider builds on the one proposed by Decamps, Mariotti, Rochet, and Villeneuve (2011) (DMRV), but with corporate and personal taxes added and with a debt-equity choice problem arising from the presence of taxation.\footnote{Besides DMRV, recent developments in dynamic models of corporate savings include Gamba and Triantis (2008), Riddick and Whited (2009), Gryglewicz (2011), Anderson and Carverhill (2012), Hugonnier, Malamud, and Morellec (2014), and Hugonnier and Morellec (2014), among others.} Thus, while their analysis focuses on the management of the firm’s asset side of the balance sheet, our analysis comprises both an asset and liability management problem. As in DMRV, we model the firm in continuous time. It has a productive asset generating \( iid \) earnings shocks. The asset costs \( K \) to set up and the entrepreneur who founds the firm must raise funds to cover both this set-up cost and the firm’s initial cash buffer. Funds can be raised by issuing equity or term debt to outside investors. The firm may also obtain a line of credit (LOC) commitment from a bank.

We show that financial constraints can make the firm’s shareholders endogenously risk-averse (equity value is \textit{concave} in the cash holdings) even in the presence of risky debt. This is in contrast to the standard risk-shifting intuition as in Jensen and Meckling (1976) and Leland (1998). Nonetheless, a conflict between equity-holders and debt-holders still arises because they tend to have different exposures to the liquidity risks and hence different degrees of effective risk aversion. This result implies that covenants can be important for the financial policies and firm value for financially constrained firms.

The financially constrained firm has multiple margins of adjustment in response to a change in its environment, including debt, equity, and cash holding, while an unconstrained firm only adjusts its debt and equity. We show that a number of surprising results on the firm’s financial policy arise from the additional cash margin; so much so that several of the key predictions from existing tradeoff theories of capital structure no longer apply for constrained firms.

Consider first the effects of a cut in the corporate income tax rate. Such a cut reduces the net tax advantage of debt and should result in a reduction in debt financing under the standard tradeoff theory. However, the main effects of the tax cut for a constrained firm are the increases in the after-tax return on corporate savings and in the volatility of
after-tax cash flows, so that it responds by increasing its cash holdings. The increase in cash holdings can be so significant that the servicing costs of debt decline to the point that they compensate for the reduced tax advantage of debt. On net, the firm can barely change its debt policy in response to the reduction in corporate tax rates.

Consider second the effects of an increase in the profitability of the productive asset. Under the standard tradeoff theory, the firm ought to issue more debt so as to shield the higher profits from taxation. The financially constrained firm not only increases its debt but also reduces its cash holding (by paying out more to shareholders), as it is able to replenish its cash stock faster due to higher profitability. In comparison, the constrained firm’s debt increase will be smaller than that of an unconstrained firm both because higher coupon raises the debt servicing costs, holding the cash policy fixed, and because the cash policy adjustment induces a further increase in the debt servicing costs. This result can help explain the empirical finding that financially constrained firms do not adjust their leverage to changes in profitability.

Third, the effects of an increase in volatility of cash flows on the financial policy are also surprising. Due to the precautionary savings motive, the financially constrained firm wants to increase its cash buffers in response to an increase in cash-flow volatility. When the (endogenously determined) marginal source of funding is debt, it is optimal for the firm to raise more debt and save the proceeds as cash. This is the opposite to the predicted effect under the standard tradeoff theory, whereby the firm responds to higher cash-flow risks by reducing leverage so as to lower expected bankruptcy costs.

The tradeoff theory of capital structure is often pitted against the pecking order theory, with numerous empirical studies seeking to test them either in isolation or in a horse race (see Fama and French (2012) for a recent example). The empirical status of the tradeoff theory has been and remains a hotly debated question. Some scholars, most notably Myers (1984), have claimed that they “know of no study clearly demonstrating that a firm’s tax status has predictable, material effects on its debt policy.” In a later review of the capital structure literature Myers (2001) further added “A few such studies have since appeared ⋅⋅⋅ and none gives conclusive support for the tradeoff theory.”
More recently, a number of empirical studies that build on the predictions of structural models in the vein of Fischer, Heinkel, and Zechner (1989), Leland (1994), Goldstein, Ju, and Leland (2001), Ross (2005), and Strebulaev (2007) by adding transaction costs when the firm changes its capital structure, have found empirical support for the dynamic tradeoff theory (see e.g., Hennessy and Whited (2005), Leary and Roberts (2005), and Lemmon, Roberts, and Zender (2008)). However, it is important to observe that in reality corporate financial decisions are not only shaped by tax-induced tradeoffs; they also depend on external financing costs and liquidity considerations. We therefore need to better understand how capital structure and other corporate financial decisions are jointly determined when the firm responds to tax incentives while simultaneously managing its cash reserves in order to relax its financial constraints, which is the goal of this paper.

As relevant as it is to analyze an integrated framework combining both tax and precautionary-savings considerations, there are, surprisingly, only a few attempts in the literature at addressing this problem. In a discrete time setting, Hennessy and Whited (2005, 2007) consider a dynamic tradeoff model for a firm facing equity flotation costs in which the firm can issue short-term debt. Riddick and Whited (2009) develop a corporate savings model and show that corporate savings and cash flow can be negatively related after controlling for $q$ when productivity shocks are persistent. DeAngelo, DeAngelo, and Whited (2011) develop and estimate a dynamic capital structure model with taxes and external financing costs of debt and show that while firms have a target leverage ratio, they may temporarily deviate from it in order to economize on debt servicing costs. A key difference of our paper from these papers is that, instead of only allowing for one-period debt (or no debt in the case of Riddick and Whited (2009)), our model introduces term debt, which can co-exist with cash and thus allows us to study the interactions between debt and cash holdings. Gamba and Triantis (2008) extend Hennessy and Whited (2005) by introducing debt issuance costs and obtain the simultaneous existence of debt and cash. They restrict long-term debt to be risk-free and thus only partially address the impact of financial constraint on the trade-off theory.\footnote{Anderson and Carverhill (2012) consider a continuos-time model of optimal cash holding policy for two exogenously given levels of debt.} We also fully characterize the firm’s cash
management policy and demonstrate the central role of the marginal value of liquidity on the financial policies of a constrained firm.

Broadly speaking, our work is also related to the dynamic optimal contracting formulation that derives corporate liquidity as optimal financial contracts in environments where not all cash flows generated by the firm are observable or verifiable.\(^4\)

A strength of our model is that it allows for a quantitative valuation of debt and equity, as well as a characterization of corporate financial policy, that can be closely linked to methodologies applied in reality, such as the adjusted present value method. Importantly, our model highlights that the classical structural credit-risk valuation models in the literature are missing an important explanatory variable: the firm’s liquidity (cash and credit) holdings, which affect both equity and debt value. Starting with Merton (1974) and Leland (1994), the standard structural credit risk models mainly focus on how shocks to asset fundamentals or cash flows affect the risk of default, but do not explicitly consider liquidity management.

2 Model

First we set up the baseline model and then provide the solution for the Miller’s benchmark.

2.1 Baseline Model Setup

A financially constrained risk-neutral entrepreneur has initial liquid wealth \(W_0\) and a valuable investment project which requires an up-front setup cost \(K > 0\) at time 0.

**Investment project.** Let \(Y\) denote the project’s (undiscounted) cumulative before-tax cash flows (profits). For simplicity, we assume that operating profits are independently and identically distributed (\(i.i.d.\)) over time and that cumulative operating profits \(Y\) follow an

\(^4\)See Bolton and Scharfstein (1990), DeMarzo and Fishman (2007), Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo and Sannikov (2006), Piskorski and Teichyti (2010), Biais, Mariotti, Rochet, and Villeneuve (2010), and Demarzo, Fishman, He, and Wang (2012). Rampini and Viswanathan (2010) develop a dynamic model of collateralized financing, in which the firm has access to complete markets but is subject to endogenous collateral constraints induced by limited commitment.
where $Z$ is a standard Brownian motion. Over a time interval $\Delta t$, the firm’s profit is normally distributed with mean $\mu \Delta t$ and volatility $\sigma \sqrt{\Delta t} > 0$. This earnings process is widely used in the corporate finance literature. For example, DeMarzo and Sannikov (2006) and Decamps, Mariotti, Rochet, and Villeneuve (2011) use the same continuous-time process (1) in their analyses.\footnote{Piskorski and Tchistyi (2010), Bolton, Chen, and Wang (2011), and Demarzo, Fishman, He, and Wang (2012) also use similar setups to model income and/or productivity shocks. Bolton and Scharfstein (1990), Hart and Moore (1994, 1998), Biais, Mariotti, Plantin, and Rochet (2007), and DeMarzo and Fishman (2007) model cash flow processes using the discrete-time counterpart of (1).} Note that the cumulative profit process (1) implies that the firm can potentially accumulate large losses over a finite time period. The project can be liquidated at any time (denoted by $T$) with a liquidation value $L < \mu / r$, where $r$ is the pre-tax risk-free rate. That is, liquidation is inefficient. To avoid or defer inefficient liquidation, the firm needs funds to cover operating losses and to meet various payments. Should it run out of liquidity, the firm either liquidates or raises new funds in order to continue operations. Therefore, liquidity can be highly valuable under some circumstances as it allows the firm to continue its profitable but risky operations.

Taxes. As in Miller (1977), DeAngelo and Masulis (1980), and the subsequent corporate taxation literature, we suppose that earnings after interest are taxed at the corporate income tax rate $\tau_c > 0$. At the personal level, income from interest payments is taxed at rate $\tau_i > 0$, and income from equity is taxed at rate $\tau_e > 0$. At the personal level, given that capital gains may be deferred, we generally expect that $\tau_e < \tau_i$ even when interest, dividend and capital gains income is taxed at the same marginal personal income tax rate.

External financing: equity, debt, and credit line. Firms often face significant external financing costs due to asymmetric information and managerial incentive problems. We do not explicitly model informational asymmetries nor incentive problems. Rather, to be able to work with a model that can be easily calibrated, we directly model the costs...
arising from informational and incentive frictions in reduced form. To begin with, we assume that the firm can only raise external funds once at time 0 by issuing equity, term debt and/or credit line, and that it cannot access capital markets afterwards. We can generalize our model by allowing the firm to repeatedly access capital markets.

As in Leland (1994) and Goldstein, Ju, and Leland (2001) we model debt as a potentially risky perpetuity issued at par $P$ with regular coupon payment $b$. Should the firm be liquidated, the debt-holders have seniority over other claimants for the residual value from the liquidated assets. In addition to the risky perpetual debt, the firm may also issue external equity. We assume that there is a fixed cost $\Phi$ for the firm to initiate external financing (either debt or equity or both). As in Bolton, Chen, and Wang (2011), equity issuance involves a marginal cost $\gamma_E$ and debt issuance involves a marginal cost $\gamma_D$.

We next turn to the firm’s liquidity policies. The firm can save by holding cash and by borrowing via the credit line. Let $W_t$ denote the firm’s liquidity holdings at time $t$. When $W_t > 0$, the firm is in the cash region. When $W_t < 0$, the firm is in the credit region.

At time 0, the firm chooses the size of its credit line $C$, which is the maximal credit commitment that the firm obtains from the bank. This credit commitment is fully collateralized by the firm’s physical capital. For simplicity, we assume that credit line is senior to term debt and the amount of credit line cannot exceed the firm’s liquidation value, $C \leq L$, so that the credit line is risk-free.

Under the terms of the credit line the firm has to pay a fixed commitment fee $\nu(C)C$ per unit of time on the (unused) amount of the credit line. We specify $\nu(C) = \eta C$ where $\eta > 0$ is the credit line commitment fee parameter. Intuitively, once it draws down an amount $|W_t| < C$ it must pay the commitment fee on the residual, $\nu(C)(C + W_t)$. The economic logic behind this cost function is that the bank providing the LOC has to either incur more monitoring costs or higher capital requirement costs when it grants a larger LOC. The firm can tap the credit line at any time for any amount up the limit $C$ after securing the credit line $C$ at time 0. For the amount of credit that the firm uses, the interest spread

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6More generally, we can have separate fixed costs $\Phi_D$ and $\Phi_E$ for debt and equity issuance, which will be equivalent to having a joint cost $\Phi$ when the firm chooses to use only one type of external financing.
over the risk-free rate $r$ is $\delta$. This spread $\delta$ is interpreted as an intermediation cost in our setting as credit is risk-free. Note that the credit line only incurs a flow commitment fee and no up-front fixed cost.

**Liquidity management: cash and credit line.** Liquidity management is at the core of our analysis. As will become clear, it is suboptimal for the firm to draw down the credit line if the firm’s cash holding is positive. Indeed, the firm can always defer using the costlier credit line option as long as it has unused cash on its balance sheet. That is, we will not have the co-existence of cash and credit line usage at the same time.

**Cash region:** $W \geq 0$. We denote by $U_t$ the firm’s cumulative (non-decreasing) after-tax payout to shareholders up to time $t$, and by $dU_t$ the incremental after-tax payout over time interval $dt$. When the firm does not pay out, $dU_t = 0$, which often happens in the model, as we will show. Distributing cash to shareholders may take the form of a special dividend or a share repurchase.\(^7\) The firm’s cash holding $W_t$ accumulates as follows in the region where the firm has a positive cash reserve:

$$dW_t = (1 - \tau_c) \left[ dY_t + (r - \lambda) W_t dt - \nu(C) C dt - b dt \right] - dU_t,$$  \hspace{1cm} (2)

where $\lambda$ is a *cash-carry cost*, which reflects the idea that cash held by the firm is not always optimally deployed. That is, the before-tax return that the firm earns on its cash inventory is equal to the risk-free rate $r$ minus a *carry cost* $\lambda$ that captures in a simple way the agency costs that may be associated with free cash in the firm.\(^8\) The firm’s cash accumulation before corporate taxes is thus given by operating earnings $dY_t$ plus earnings from investments $(r - \lambda) W_t dt$ minus the credit line commitment fee $\nu(C) C dt$ minus the

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\(^7\)A commitment to regular dividend payments is suboptimal in our model. For simplicity we assume that the firm faces no fixed or variable payout costs. These costs can, however, be added at the cost of a slightly more involved analysis.

\(^8\)This assumption is standard in models with cash. For example, see Kim, Mauer, and Sherman (1998) and Riddick and Whited (2009). Abstracting from any tax considerations, the firm would never pay out cash when $\lambda = 0$, since keeping cash inside the firm then incurs no opportunity costs, while still providing the benefit of a relaxed financing constraint. If the firm is better at identifying investment opportunities than investors, we would have $\lambda < 0$. In that case, raising funds to earn excess returns is potentially a positive NPV project. We do not explore cases in which $\lambda < 0$. 

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interest payment on term debt $bdt$. The firm pays a corporate tax rate $\tau_c$ on these earnings net of interest payments and retains after-tax earnings minus the payout $dU_t$. Finally, note that the corporate tax rate $\tau_c$ lowers both the drift and the volatility of the liquidity accumulation process (2).\(^9\)

An important simplifying assumption implicit in this cash accumulation equation is that profits and losses are treated symmetrically from a corporate tax perspective. In practice losses can be carried forward or backward only for a limited number of years, which introduces complex non-linearities in the after-tax earnings process. As Graham (1996) has shown, in the presence of such non-linearities one must forecast future taxable income in order to estimate current-period effective tax rates. To avoid this complication we follow the literature in assuming that after-tax earnings are linear in the tax rate (see e.g., Leland (1994) and Goldstein, Ju, and Leland (2001). Hennessy and Whited (2005) provide a non-linear tax schedule treatment.

**Credit region: $W \leq 0$.** In the credit region, credit $W_t$ evolves similarly as $W_t$ does in the cash region, except for changes resulting from the fact that in this region the firm is partially drawing down its credit line and paying interest at the rate of $r + \delta$:

$$
\frac{dW_t}{dt} = (1 - \tau_c) [dY_t + (r + \delta)W_tdt - \nu(C)(W_t + C)dt - bdt] - dU_t, 
$$

where $\delta$ denotes the interest rate spread over the risk-free rate, and the commitment fee is charged on the unused LOC commitment $W_t + C$. If the firm exhausts its maximal credit capacity, so that $W_t = -C$, it has to either close down and liquidate its assets or raise external funds to continue operations. In the analysis of our baseline model, we assume that the firm will be liquidated if it runs out of all available sources of liquidity including both cash and credit line. One may generalize our model by granting the firm the option to raise new funds through external financing.

\(^9\)The tax implications on the volatility of after-tax labor income have first been explored by Kimball and Mankiw (1989) in a precautionary savings model for households.
Optimality. We solve the firm’s optimization problem in two steps. Proceeding by backward induction, we consider first the firm’s *ex post* optimization problem after the initial capital structure (external equity, debt, and credit line) has been chosen. Then, we determine the *ex ante* optimal capital structure.

The firm’s *ex post* optimization problem. The firm chooses its payout policy $U$ and liquidation timing $T$ to maximize the ex post value of equity subject to the liquidity accumulation equations (2) and (3):\(^{10}\)

$$
\max_{U,T} \mathbb{E} \left[ \int_0^T e^{-r(1-\tau_i)t} dU_t + e^{-r(1-\tau_i)T} \max\{L_T + W_T - P - G_T, 0\} \right].
$$

(4)

Note that $P$ denotes the proceeds from the debt issue. The first term in (4) is the present discounted value of payouts to equity-holders until stochastic liquidation, and the second term is the expected liquidation payoff to equity-holders. Here, $G_T$ is the tax bill for equity-holders at liquidation. It is possible that equity-holders realize a capital gain upon liquidation. In this event liquidation triggers capital gains taxes for them. Capital gains taxes at liquidation are given by:

$$
G_T = \tau_e \max\{W_T + L_T - P - (W_0 + K), 0\}.
$$

(5)

Note that the basis for calculating the capital gain is $W_0 + K$, the sum of liquid and illiquid initial asset values, and importantly, $W_0$ is endogenously determined. Let $E(W_0)$ denote the value function (4).

The *ex ante* optimization problem. What should the firm’s initial cash holding $W_0$ be? And in what form should $W_0$ be raised? The firm’s financing decision at time 0 is to jointly choose the initial cash holding $W_0$, the line of credit with limit $C$, and the optimal capital structure (debt and equity). Specifically, the entrepreneur chooses any combination of (i) a perpetual debt issue with coupon $b$, (ii) a credit line with limit $C$, and (iii) an equity issue of a fraction $a$ of total shares outstanding.

\(^{10}\)Note that this objective function does not take into account the benefits of cash holdings to debt-holders. We later explore the implications of constraints on equity-holders’ payout policies that might be imposed by debt covenants.
There is a positive fixed cost $\Phi > 0$ in tapping external financial markets, so that securities issuance is lumpy. We also assume that there is a positive variable cost in raising debt ($\gamma_D \geq 0$) or equity ($\gamma_E \geq 0$). Let $F$ denote the proceeds from the equity issue. We focus on the economically interesting case where some amount of external financing is optimal. After paying the set-up cost $K > 0$, and the total issuance costs ($\Phi + \gamma_DP + \gamma_EF$) the firm ends up with an initial cash stock of:

$$W_0 = W_{0-} - K - \Phi + (1 - \gamma_D)P + (1 - \gamma_E)F,$$  \hspace{1cm} (6)

where $W_{0-}$ is the entrepreneur’s initial cash endowment before financing at time 0.

The entrepreneur’s *ex ante* optimization problem is to maximize the ex ante project value by solving

$$U_0 = \max_{a,b,C} (1 - a)E(W_0; b, C) - W_{0-},$$  \hspace{1cm} (7)

where $E(W)$ is the solution of (4), $U_0$ denotes the maximal value obtained from the optimization problem defined by (7), and where the following competitive pricing conditions for debt and equity must hold:

$$P = D(W_0),$$  \hspace{1cm} (8)

and

$$F = aE(W_0).$$  \hspace{1cm} (9)

Before formulating debt value $D(W_0)$ and equity value $E(W_0)$ as solutions to differential equations and proceeding to characterize the solutions to the ex post and ex ante optimization problems we begin by describing the classical *Miller irrelevance solution* in our model for the special case where the firm faces no financing constraints.

### 2.2 The Miller Benchmark

Miller (1977) observed that as long as the net tax benefit of debt is strictly positive, firms should only rely on debt financing. He then derived an equilibrium where the aggregate amount of debt in the economy is tied down by the condition that in equilibrium the
marginal investor has personal income tax rates, $\tau_i$ and $\tau_e$, such that for that investor there is no net tax advantage of debt. Similarly in our setup, we will show that under the Miller benchmark where the firm faces neither external financing costs ($\Phi = \gamma_P = \gamma_F = \eta = \delta = 0$) nor any cash carry cost ($\lambda = 0$), the firm will rely exclusively on debt financing as long as there is a net tax advantage of debt.

Without loss of generality we will take it that in this idealized world the firm never relies on a credit line and simply issues new equity if it is in need of cash to service the term debt. Given that shocks are i.i.d. the firm then never defaults. Miller (1977) argues that the effective tax benefit of debt, which takes into account both corporate and personal taxes, is

$$\tau^* = \frac{(1 - \tau_i) - (1 - \tau_c)(1 - \tau_e)}{1 - \tau_i} = 1 - \frac{(1 - \tau_c)(1 - \tau_e)}{(1 - \tau_i)}. \quad (10)$$

For a firm issuing a perpetual interest-only debt with coupon payment $b$, its ex post equity value is then:

$$E^* = E \left[ \int_0^\infty e^{-r(1-\tau_i)t} (1 - \tau_c)(1 - \tau_e)(dY_t - bdt) \right] = \frac{1}{r} (1 - \tau^*) (\mu - b). \quad (11)$$

For a perpetual debt with no liquidation ($T = \infty$), ex post debt value is simply $D^* = b/r$ as both the after-tax coupon and the after-tax interest rate are proportional to before-tax coupon $b$ and before-tax interest rate $r$ with the same coefficient $(1 - \tau_i)$.

The firm’s total value, denoted by $V^*$, is given by the sum of its debt and equity value:

$$V^* = E^* + D^* = \frac{\mu}{r} (1 - \tau^*) + \frac{b}{r} \tau^*, \quad (12)$$

where the first term is the value of the unlevered firm and the second term is the present value of tax shields. First, as long as $\tau^* > 0$, (12) implies that the optimal leverage for a financially unconstrained firm is the maximally allowed coupon $b$. Given that the firm cannot borrow more than its value (or debt capacity), it may pledge at most 100% of its cash flow by setting $b^* = \mu$. In this case, firm value satisfies the familiar formula $V^* = \mu/r$. As we will show, for a financially constrained firm, even with $\tau^* > 0$, liquidity
considerations will lead the firm to choose moderate leverage.

3 Model Analysis

We now characterize the solutions to the ex post and ex ante problems for the firm. As we will show below, the firm pays out when its cash stock $W$ exceeds an endogenous upper barrier $\bar{W}$, and liquidates itself once it exhausts its LOC (when $W = -C$). In the interior region, $-C \leq W \leq \bar{W}$, the firm services interest payments for its term debt and accumulates liquidity.

3.1 Optimal Payout Policy and the Value of Debt and Equity

To determine whether it is ever optimal for the firm to distribute cash to the shareholders, consider the following two payout strategies for a financially unconstrained firm. First, suppose the firm pays out a dollar of cash today, which will be taxed at the equity income tax rate $\tau_e$. Then, the interest income on the personal savings over a small time interval $\Delta$, $r(1 - \tau_e)\Delta$, will be taxed at the personal interest income tax rate $\tau_i$. Second, suppose the firm delays the payout of the dollar for an extra period $\Delta$. In this case, the interest income on corporate savings will be $(r - \lambda)\Delta$, which is taxed at the corporate income tax rate $\tau_c$ and then at the rate $\tau_e$ when it is paid out to the shareholders. A comparison of the after-tax values of the two strategies above gives the following necessary and sufficient condition for the firm to eventually payout its cash:

$$(r - \lambda)(1 - \tau_c) < r(1 - \tau_i).$$

(13)

Condition (13) shows that, when $\tau_c > \tau_i$, there will be a tax-induced cost for carrying cash inside firm. As a result, the firm will not accumulate unlimited cash holdings even in the absence of direct cash-carry cost ($\lambda = 0$).

If the firm is financially constrained, i.e., if there is a wedge between the internal and external costs of funds, then holding cash inside the firm has the additional benefit of
reducing the risk of costly liquidation or costly external financing, which are not taken into account in condition (13). The payout boundary for the constrained firm is optimally chosen by equity-holders to trade off the after-tax efficiency of personal savings versus the expected costs of premature liquidation when the firm runs out of cash, which we characterize next.

Let $E(W)$ denote the after-tax value of equity. In the interior cash hoarding region $0 \leq W \leq W$ and the interior credit region, $-C \leq W \leq 0$, standard risk-neutral pricing arguments imply that $E(W)$ satisfies a pair of ODEs (A3-A4) given in the appendix.

At the endogenous payout boundary $W$, equity-holders must be indifferent between retaining cash inside the firm and distributing it to shareholders, which implies

$$E'(W) = 1 - \tau_e.$$  \hfill (14)

In addition, since equity-holders optimally choose the payout boundary $W$, the following super-contact condition must also be satisfied:

$$E''(W) = 0.$$  \hfill (15)

Substituting (14) and (15) into the ODE for $E(W)$ (A3), we then obtain the value of equity at the payout boundary $W$:

$$E(W) = \frac{(1 - \tau^*) \left( \mu - b + (r - \lambda)W - \nu(C)C \right)}{r},$$  \hfill (16)

where $\tau^*$ is the Miller tax rate given by (10). The expression (16) can be interpreted as a “steady-state” perpetuity valuation by slightly modifying the Miller formula (11) with the added term $(r - \lambda)W - \nu(C)C$, which adjusts for the interest income on the maximal corporate cash holdings $W$ and the commitment fee on the unused credit line.

At the left boundary $W = -C$, the firm is liquidated. Here, equity value is given by

$$E(-C) = \max \{0, L - C - P - G\},$$  \hfill (17)
where $G$ denotes the capital gains taxes given in (5) at the moment of exit. There are two scenarios to consider. First, term debt is fully repaid at liquidation thus being risk-free. If debt is risky, the seniority of debt over equity implies that equity is worthless, so that $E(−C) = 0$. Recall that the credit line is always fully repaid.

Let $D(W)$ denote the after-tax value of debt. Similar to $E(W)$, $D(W)$ satisfies a pair of ODEs (A5-A6) given in the appendix. The boundary conditions at $W = −C$ and $W = W$ are:

\begin{align}
D(−C) &= \min \{L − C, P\}, \\
D'(W) &= 0.
\end{align}

Condition (18) follows from the absolute priority rule, which states that debt payments have to be serviced in full before equity-holders collect any liquidation proceeds. Condition (19) follows from the fact that the expected life of the firm does not change as $W$ approaches $W$ (since $W$ is a reflecting boundary).

For a given payout boundary $W$, the value of equity $E(W)$ can be solved analytically (in the form of confluent hypergeometric functions) via the ODEs (A3-A4) with the boundary conditions (14) and (17). Similarly, the value of debt $D(W)$ can also be solved analytically with the boundary conditions (18) and (19). One can then solve for the optimal payout boundary via (15).

**Firm value $V(W)$ and enterprise value $Q(W)$**. Since debt-holders and equity-holders are the firm’s two claimants and credit line use is default-free and is fully priced in the equity value $E(W)$, we define the firm’s total value $V(W)$ as the sum of equity and term debt, $V(W) = E(W) + D(W)$. Following the standard practice in both academic and industry literatures, we define enterprise value $Q(W)$ as firm value netting out cash, $Q(W) = V(W) − W$. Note that $Q(W)$ is purely an accounting definition and may not be very informative about the economic value of the productive asset under financial constraints. For example, one discrepancy is that while firm value $V(W)$ is after-tax, the cash value $W$ is before-tax.
Having characterized the market values of debt and equity, we now turn to the firm’s ex ante optimization problem, which involves the choice of an optimal ‘start-up’ cash reserve $W_0$, an optimal credit line commitment with limit $C$, and an optimal debt and equity structure.

### 3.2 Optimal Capital Structure

At time 0, the entrepreneur chooses the fraction of outside equity $a$, the coupon on the perpetual risky debt $b$, and the credit line limit $C$ (with implied $W_0$) to maximize the initial value of inside equity as specified by (7), subject to the budget constraint (6).

Without loss of generality we assume that the risk-neutral entrepreneur has no wealth before setting up the firm, $W_{0-} = 0$. The optimal amount of initial cash holding $W_0$ the firm starts out with (after the time-0 financing arrangement) is then given by the solution to the following equation, which defines a fixed point for $W_0$:

$$
(1 - \gamma_E) aE(W_0; b, C) + (1 - \gamma_D) D(W_0; b, C) = W_0 + K + \Phi.
$$

(20)

There can be zero, one, or two solutions to the above fixed-point problem. In the case of zero solution, the firm simply cannot finance the project at $t = 0$. The case of two solutions represents two equilibria, one with a higher level of $W_0$, one lower. The intuition for the possibility of multiple equilibria is that outside investors can give the firm’s debt and equity a high (low) valuation based on the belief that the firm has a strong (weak) liquidity position, which means that the firm can raise a larger (smaller) amount of funds for the same choice of $(a, b, C)$. The resulting initial cash holding $W_0$ will be high (low), thus justifying the belief of the investor. Using the concavity of $D(W)$ and $E(W)$, one can show that only the equilibrium with higher $W_0$ is stable, which is what we will focus on in our analysis.

The entrepreneur is juggling with the following considerations in determining the firm’s optimal capital structure. By raising funds through term debt instead of external equity, the entrepreneur can obtain a tax shield and hold onto a larger fraction of equity ownership.
A cost of debt financing is that the perpetual interest payments $b$ must be serviced out of the firm’s stock of cash or credit line, thus draining its liquidity. To reduce the risk of running out of cash, the entrepreneur can start the firm with a larger cash holding $W_0$ (by issuing more debt and/or equity), delay payout (by choosing a larger $W$), and take out a larger LOC commitment $C$. The costs of these measures are that the entrepreneur will pay a larger issuance cost at time 0, keep a smaller equity ownership, incur higher (direct and indirect) cash-carry costs, and pay more for the commitment fees and interest spreads on the credit line. In addition, since credit line is senior to term debt, a larger $C$ can reduce the ex-ante valuation of term debt.

Depending on underlying parameter values, the firm’s time-0 optimal capital structure can admit three possible solutions: (i) no term debt (equity issuance only, with possibly an LOC); (ii) term debt issuance only (with, again, a possible LOC); and (iii) a combination of equity and term debt issuance (with a possible LOC).

4 Quantitative Results

Parameter values and calibration We choose the model parameters as follows. First, we set the corporate income tax rate at $\tau_c = 35\%$ as in Leland (1994), and the two personal tax rates on equity income and interest income at $\tau_e = 12\%$ and $\tau_i = 30\%$ as in Hennessy and Whited (2007). The tax rate $\tau_e$ on equity income reflects the fact that the taxation of capital gains can be deferred until capital gains are realized. Based on our assumed tax rates, the Miller effective tax rate as defined in (10) is $\tau^* = 18.3\%$.

Second, we set the annual risk-free rate to $r = 6\%$. The mean and volatility of annual return on capital is set to $\mu = 12\%$ and $\sigma = 10\%$ based on the estimates of Acharya, Almeida, and Campello (2013) and Sufi (2009). For the interest spread on the LOC, we choose $\delta = 0.25\%$ to capture the costs for banks to monitor the firm (there is no default risk for the LOC). For the LOC commitment fee, we calibrate $\eta = 0.028$ to match the average (unused) LOC-to-asset ratio ($C = 0.10$) in Sufi (2009). The implied average proportional commitment fee is 28 basis points, close to the median commitment fee of 25
Table 1: **Parameters.** This table reports the parameter values for the baseline model. All the parameter values are annualized when applicable.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>$r$ 6%</td>
</tr>
<tr>
<td>Mean return on capital</td>
<td>$\mu$ 12%</td>
</tr>
<tr>
<td>Volatility of return on capital</td>
<td>$\sigma$ 10%</td>
</tr>
<tr>
<td>Initial investment</td>
<td>$K$ 1</td>
</tr>
<tr>
<td>Liquidation value</td>
<td>$L$ 0.9</td>
</tr>
<tr>
<td>Tax rate on corporate income</td>
<td>$\tau_c$ 35%</td>
</tr>
<tr>
<td>Tax rate on equity income</td>
<td>$\tau_e$ 12%</td>
</tr>
<tr>
<td>Tax rate on interest income</td>
<td>$\tau_i$ 30%</td>
</tr>
<tr>
<td>Fixed financing cost</td>
<td>$\Phi$ 1%</td>
</tr>
<tr>
<td>Prop. debt financing cost</td>
<td>$\gamma_D$ 1%</td>
</tr>
<tr>
<td>Prop. equity financing cost</td>
<td>$\gamma_E$ 6%</td>
</tr>
<tr>
<td>Credit line spread</td>
<td>$\delta$ 0.25%</td>
</tr>
<tr>
<td>LOC commitment fee parameter</td>
<td>$\eta$ 2.8%</td>
</tr>
</tbody>
</table>

basis points as documented by Sufi (2009).

Third, regarding the external financing costs, we take the fixed cost $\Phi$ to be 1% of capital stock $K$ as in Bolton, Chen, and Wang (2011), and we take the marginal debt issuance cost $\gamma_D = 1\%$ and the marginal equity issuance cost $\gamma_E = 6\%$ based on the empirical findings of Altinkilic and Hansen (2000).

Fourth, we set the cash-carrying cost to $\lambda = 0.5\%$ due to agency or governance factors. Finally, the liquidation value of capital is $L = 0.9$ as in Hennessy and Whited (2007). Table 1 summarizes all the parameter values.

### 4.1 Optimal Capital Structure

Based on our baseline parameter values, the optimal coupon is $b = 0.08$, and there is no outside equity stake, $a = 0$. Moreover, the entrepreneur obtains a LOC commitment of $C = 0.10$, starts the firm with a cash buffer of $W_0 = 0.303$, and sets the payout boundary at $\bar{W} = 0.326$, all of which contribute to reducing liquidity risk. The entrepreneur obtains an initial equity value of $U_0 = 0.763$ under the optimal capital structure. These results and information about financial leverage are reported in Table 2.

**The Ex-Post Value of Equity and Debt** Figure 1 plots the value of equity $E(W)$ in Panel A and the marginal equity value of liquidity $E'(W)$ in Panel B. When $W$ reaches the endogenous lower boundary $\underline{W} = -C = -0.10$, the firm has run out of its maximal
Table 2: **Optimal capital structure.** This table reports the optimal capital structure results from the baseline case.

<table>
<thead>
<tr>
<th>credit line rate</th>
<th>coupon rate</th>
<th>outside equity</th>
<th>payout boundary</th>
<th>initial cash value</th>
<th>project value</th>
<th>debt value</th>
<th>interest coverage</th>
<th>market leverage</th>
<th>$\frac{D_0}{D_0+E_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$b$</td>
<td>$a$</td>
<td>$\bar{W}$</td>
<td>$W_0$</td>
<td>$U_0$</td>
<td>$D_0$</td>
<td>$\mu/b$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.080</td>
<td>0</td>
<td>0.326</td>
<td>0.303</td>
<td>0.763</td>
<td>1.326</td>
<td>1.497</td>
<td>0.635</td>
<td></td>
</tr>
</tbody>
</table>

liquidity supply and is liquidated. At that point equity is worthless, because the asset liquidation value $L = 0.9$ is lower than the face value of debt $D_0 = 1.326$. When $W$ hits $\bar{W} = 0.326$, it is optimal for the firm to pay out any cash in excess of $\bar{W}$. Indeed, at that point the marginal value of liquidity inside the firm for equity-holders is equal to the after-tax value of a dollar of payout: $E'(\bar{W}) = 1 - \tau_e = 0.88$, as shown in Panel B. In the interior region, equity value $E(W)$ increases with $W$ with a slope $E'(W) > 1 - \tau_e$, reflecting the value of a higher cash buffer as insurance against the risk of early liquidation. Panel B shows that, when the firm is close to running out of cash, the marginal value of one dollar of cash to equity holders can exceed six dollars.

Remarkably, $E(W)$ is concave in the *cash holding* $W$ even though the firm is levered with *risky* debt, which appears to contradict the standard intuition of Merton (1974) that the equity of a levered firm is a call option on the firm’s assets and hence the its value should be convex in the *asset value*. This result is due to the fact that the asset value in our model is itself endogenously concave in the cash holding due to the threat of costly liquidation. In our baseline model, default is driven solely by diffusive liquidity shocks. The fact that liquidation generates losses for both debt-holders and equity holders makes the equity holders effectively risk averse and engage in precautionary corporate savings.\(^\text{11}\)

In contrast, the presence of large liquidity shocks can generate convexity in the equity value, which we show in Section 6.

The effective risk aversion of equity-holders induced by liquidity risk helps explain the risk management policies of financially constrained firms in practice. Rauh (2009) finds\(^\text{11}\)

\(^{11}\)The concavity of the equity value is easiest to see in the case without uncertainty, where the equity value will be linearly increasing in cash holding when $W > -C$ but drops discretely at $W = -C$.\(^\text{20}\)
that firms with more poorly funded pension plans tend to hold more safe assets in their portfolios, which is opposite to the prediction of the risk-shifting theory of Jensen and Meckling (1976) but consistent with our model prediction.

Panels C and D of Figure 1 show the property of debt. The market value of debt $D(W)$ is increasing and concave in $W$ (Panel C). The marginal debt value of liquidity $D'(W)$ is highly sensitive to $W$ (Panel D). $D'(W)$ can be as high as 7 near the liquidation boundary, but it approaches zero as $W$ increases towards the payout boundary $\overline{W}$, indicating that debt becomes insensitive to changes in cash holding when a firm is relatively unconstrained.
An immediate consequence of the fact that the marginal debt value of liquidity can either exceed or fall below 1 is that net debt, $D(W) - W$, will be non-monotonic in $W$. Using a static model, Acharya, Almeida, and Campello (2007) argue that cash is not equivalent to negative debt in that issuing debt and hoarding the proceeds in cash has different implications for future investment than preserving the debt capacity. Our results on net debt corroborate theirs in a dynamic setting.

Through debt value $D(W)$, we can define the credit spread as the difference between the yield on long-term debt and the risk-free rate, $S(W) = b/D(W) - r$. It is easy to see that the spread will be monotonically decreasing in cash holding $W$. Under the baseline calibration, when the firm is close to exhausting its maximal liquidity supply, the annualized credit spread exceeds 400 basis points. On the other extreme, the spread drops to merely 5 basis points as $W$ approaches the endogenous payout boundary.

Comparing the results for equity and debt, we see that while both equity value and debt value are concave in $W$, the effective risk aversion of equity- and debt-holders will generally differ due to their different exposures to the risk of liquidation. For example, equity-holders will be more concerned with liquidity risk than debt-holders when the level of term debt is low ($b$ is small), but the opposite will be true when debt level becomes sufficiently high. As a result, conflict between equity-holders and debt-holders still arises, but the policies (on cash management, financing, payout, etc.) that are optimal to equity-holders can be too conservative or too aggressive from the point of view of debt-holders. Thus, it can be in the interest of debt-holders to impose debt covenants that limit equity holders’ ability to control risk or pay out dividends ex post.

Finally, Panels E and F of Figure 1 plot the enterprise value $Q(W) = V(W) - W$ and the marginal enterprise value of liquidity $Q'(W)$. The marginal enterprise value of liquidity shows the full effect of adding liquidity into a financially constrained firm. An extra dollar of liquidity can bring a net increase in firm value that exceeds 13 dollars.

4.2 Net Tax Benefit of Debt for a Financially Constrained Firm

Miller (1977) provides a simple formula of the net tax benefit of debt for an unconstrained
firm, which nets out the tax benefit of debt at firm level against the tax disadvantage of debt at individual level. Most dynamic capital structure models that adopt a simple tax environment assume that the Miller tax rate $\tau^*$ captures the combined effects of the three tax rates (corporate, personal equity, and personal interest incomes) on leverage choices. However, in our model with financing frictions and cash accumulation, the Miller tax rate $\tau^*$ is no longer sufficient to capture the effects of corporate and personal equity/debt tax rates. The reason is that it is often optimal for a financially constrained firm to hoard cash rather than immediately pay out its earnings, which separates the time when the firm earns its profit from the time when it pays out its earnings.

Consider how an extra dollar in income generated inside the firm may be used. A marginal $\Delta$-dollar increment in income can be used in one of the following three ways: (i) paid out to debt-holders as interest, (ii) paid out to equity-holders as dividend, or (iii) retained inside the firm to increase the liquidity reserve. The after-tax interest income to debt-holders is $(1 - \tau_i)\Delta$ (not taxed at the rate $\tau_c$), while the after-tax dividend income to equity holders is $(1 - \tau_c)(1 - \tau_e)\Delta$. If the amount $\Delta$ is retained, the firm’s cash reserve will increase by $(1 - \tau_c)\Delta$, resulting in an approximate after-tax increase in equity value $E'(W_t)(1 - \tau_c)\Delta$ for small $\Delta$.

In the absence of external financing costs there is no need to retain cash. The marginal net tax benefit of debt is then based on the comparison between choices (i) and (ii), which yields the effective Miller tax rate in (10). In the presence of external financing costs, the firm prefers to retain cash instead of paying it out whenever $W_t$ is away from the endogenous payout boundary, $W_t < \overline{W}$. The marginal net tax benefit of debt then becomes

$$\tau^*(W_t) = \frac{(1 - \tau_i)\Delta - (1 - \tau_c)E'(W_t)\Delta}{(1 - \tau_i)\Delta} = 1 - \frac{(1 - \tau_c)E'(W_t)}{(1 - \tau_i)}.$$  \hspace{1cm} (21)

In other words, for a financially constrained firm the payout choice (ii), and hence the Miller formula for the net tax benefit of debt, is only relevant when a firm is indifferent between paying out and retaining cash inside the firm, i.e., when $W_t = \overline{W}$. Note that since $E'(\overline{W}) = 1 - \tau_e$, the right-hand side of (21) reduces to Miller’s effective tax rate in (10) at the payout boundary $\overline{W}$. 

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Equation (21) shows that the marginal net tax benefit of debt depends crucially on the financing constraint reflected by the marginal equity value of liquidity $E'(W)$. The marginal net tax benefit is reduced when the firm’s precautionary savings motive is high, i.e., when $E'(W)$ is high. As Figure 2 shows, the size of this effect on the net tax benefit of debt can be quite large when the firm’s cash holdings are low. The marginal net tax benefit turns negative for $W < 0.13$, and can be as low as $-4.91$ when the firm is close to running out of cash, compared to 0.183 in the Miller formula.

5 Impact of Liquidity on Capital Structure

The results in Section 4 show that a financially constrained firm will only exploit the tax advantage of debt to a limited extent. For example, when it is close to the endogenous payout boundary $W$, our financially constrained firm’s market leverage $L(W)$ is about 40% lower than the optimal market leverage for an unconstrained firm.

Besides debt conservatism, our model generates several other distinct predictions about
the capital structure. Unlike for the classical dynamic tradeoff theory where the firm’s only margin of adjustment is debt versus equity financing, in our model a financially constrained firm has an additional margin of adjustment: liquidity management. This includes cash holding, timing of payout and external financing, and credit line commitment. Thus, predictions from the standard tradeoff theory, such as how leverage responds to changes in tax rates or firms’ cash-flow characteristics may no longer hold in our model due to the fact that the constrained firm will now be concerned with the impact that changes in financial policies have on liquidity. We examine these questions in this section.

5.1 Corporate Financial Policy and Taxation

We first examine a firm’s financial policy under three tax policy scenarios that are different from the baseline model. In the first scenario, we lower the corporate tax rate from $\tau_c = 35\%$ to 25\% while keeping the other parameters fixed. The effects of this change on corporate financial policy are reported in the second row of Table 3. Cutting the corporate tax rate substantially reduces the Miller tax rate $\tau^*$ from 18.3\% to 5.7\%. Based on the intuition of the classical tradeoff model, one would expect such a decline to significantly lower the firm’s reliance on debt financing. However, in our model the firm barely changes its reliance on term debt, with the coupon $b$ dropping from 0.08 to 0.077. This is line with the findings of Heider and Ljungqvist (2013), who show that financially constrained firms do not significantly lower their debt following a reduction of the corporate tax rate.

Instead of significantly reducing its term debt, the main change in response to the lower corporate tax rate concerns the firm’s cash management. A lower corporate tax rate raises the firm’s after-tax return on its savings $(r - \lambda)(1 - \tau_c)$ from 3.575\% to 4.125\%. It also raises the volatility of the after-tax cash flows $(1 - \tau_c)\sigma$ from 7.8\% to 9\%. Moreover, since the amount of term debt issued is lower, equity-holders become more exposed to liquidity risk ex post. All these changes will encourage the firm (who acts in the interest of equity-holders) to increase its cash savings. While the initial cash holding falls from $W_0 = 0.303$ to $W_0 = 0.258$ as a result of the lower amount of debt issued, the payout boundary shifts upward significantly, from $\overline{W} = 0.326$ to 0.507.
Table 3: **Comparative statics: tax rates.** This table reports the results from comparative statics on different tax rates.

<table>
<thead>
<tr>
<th></th>
<th>Miller tax rate</th>
<th>credit line</th>
<th>coupon rate</th>
<th>payout bound</th>
<th>initial cash</th>
<th>project value</th>
<th>debt value</th>
<th>interest coverage</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau^* )</td>
<td>18.3%</td>
<td>0.100</td>
<td>0.080</td>
<td>0.326</td>
<td>0.303</td>
<td>0.763</td>
<td>1.326</td>
<td>1.497</td>
<td>0.635</td>
</tr>
<tr>
<td>( \tau_c = 25% )</td>
<td>5.7%</td>
<td>0.097</td>
<td>0.077</td>
<td>0.507</td>
<td>0.258</td>
<td>0.887</td>
<td>1.281</td>
<td>1.557</td>
<td>0.591</td>
</tr>
<tr>
<td>( \tau_e = 0% )</td>
<td>7.1%</td>
<td>0.100</td>
<td>0.080</td>
<td>0.326</td>
<td>0.303</td>
<td>0.867</td>
<td>1.326</td>
<td>1.497</td>
<td>0.605</td>
</tr>
<tr>
<td>( \tau_i = 15% )</td>
<td>32.7%</td>
<td>0.098</td>
<td>0.086</td>
<td>0.288</td>
<td>0.661</td>
<td>0.960</td>
<td>1.688</td>
<td>1.393</td>
<td>0.637</td>
</tr>
</tbody>
</table>

Therefore, should a tax reform to cut the corporate tax rate be introduced in the US, our model predicts that corporations would substantially increase their cash holdings. The other changes that such a tax reform would induce is a small reduction in the reliance on LOC (from \( C = 0.10 \) to \( C = 0.097 \)) due to the fact that cash holding has become less expensive. Naturally, the reduction in corporate taxation will also result in higher equity value and the enterprise value. The market value of debt falls due to the smaller coupon, and the credit spread is lower for most levels of cash holding.

In the second scenario, we eliminate the personal tax rate on equity income, i.e., changing \( \tau_e \) from 12\% to 0\%. The effects of this change are reported in the third row of Table 3. This change again significantly reduces the Miller tax rate, with \( \tau^* \) dropping from 18.3\% to 7.1\%, but the firm’s payout policy remains unchanged. While the reduction in equity income tax \( \tau_e \) does raise the equity and enterprise value, it has no effect on the firm’s ex ante financial decisions (amount of term debt issued, line of credit, the initial cash holding, and equity issue) because \( 1 - \tau_e \) appears in the ex ante optimization problem as a multiplicative constant. Moreover, the firm’s ex post cash policy is also invariant to \( \tau_e \).

In the third scenario, we lower the personal tax rate on interest income, \( \tau_i \), from 30\% to 15\%.\(^\text{12}\) The results are reported in the fourth row of Table 3. The cut in \( \tau_i \) almost doubles the Miller tax rate \( \tau^* \) from 18.3\% to 32.7\%. As indicated by equation (21), a

\(^\text{12}\)Since \( r \) is a pre-tax risk-free rate, we adjust \( r \) to keep the after-tax risk-free rate \( (1 - \tau_i) r \) fixed while changing \( \tau_i \).
lower $\tau_i$ does directly increases the net tax benefit of debt, which results in an increase in the coupon rate from $b = 0.08$ to $0.086$. The higher coupon combined with the reduction in $\tau_i$ raises the value of term debt $D(W)$, which more than doubles the initial cash holding from $W_0 = 0.303$ to $0.661$.

While the firm raises more external funds by issuing more debt, it actually chooses to hold less cash on average, as indicated by the drop in the payout boundary from $\bar{W} = 0.303$ to $0.288$. This is because the higher level of debt in place reduces equity-holders’ exposure to the risk of liquidation, who are willing to hold less cash in exchange for a somewhat higher short-term payout. The result is that there will be an immediate payout of $W_0 - \bar{W} = 0.373$ to the equity-holders after raising external funds.

### 5.2 Profitability, Earnings Volatility, and Financial Policy

Next, we examine how the firm’s optimal financial policy changes with profitability $\mu$ and cash-flow volatility $\sigma$.

The standard tradeoff theory predicts a positive relation between profitability and financial leverage. Simply put, higher profitability allows the firm to require a higher tax shield, which is obtained by committing to higher interest payments. Remarkably, this seemingly obvious prediction is not borne out for a financially constrained firm. As shown in the second row of Table 4, when we increase the mean return to capital $\mu$ from $12\%$ to $14\%$, the firm keeps its LOC commitment unchanged at $C = 0.10$ and mainly adjusts
its coupon from \( b = 0.08 \) to 0.094, and its payout boundary \( W \) from 0.326 to 0.313. In other words, the firm raises more debt to take advantage of the tax shield, but can afford to reduce its cash holding thanks to a higher profitability \( \mu \). The net effect is that the market leverage, instead of rising, drops from 0.635 to 0.599.

Another well known prediction from the standard tradeoff theory is that an increase in cash-flow risk reduces leverage. Riskier firms are expected to reduce their indebtedness because they face higher expected bankruptcy costs. To examine this prediction, we raise the volatility of cash flows from \( \sigma = 10\% \) in the baseline case to 20\%. The results are reported in the third row of Table 4. In this context it is striking to observe that the amount of term debt the firm issues actually increases, with the coupon \( b \) rising from 0.08 to 0.086, and the interest coverage dropping from 1.497 to 1.391.

With high cash-flow risk, the firm also chooses to issue outside equity in addition to term debt, with the outside equity share increasing from \( a = 0 \) in the baseline case to 0.205. The purpose of issuing more debt as well as outside equity is to build up a sufficient initial cash buffer \( W_0 \), which substantially increases from 0.303 to 0.506. Moreover, the firm adopts a more conservative payout policy, with the endogenous payout boundary \( \bar{W} \) jumping from 0.326 to 0.774, which implies a substantial increase in the average corporate savings. Overall, an increase in volatility is bad news for the firm, as witnessed by the decline in ex-ante project value from 0.763 to 0.633. Intuitively, the firm attempts to make up for this worsening situation by building a larger cash reserve to reduce the probability of liquidation, which requires more external financing, including more debt and equity. In sum, the main lesson emerging from this comparative statics exercise is that the observation of higher debt and leverage for riskier firms is not necessarily a violation of the tradeoff theory. It points to the importance of incorporating into the standard model a precautionary savings motive.

In Figure 3, we explore more systematically how the firm’s financial policy varies with cash-flow volatility \( \sigma \). This exercise demonstrates remarkable richness in the ways firms adjust their financial decisions in response to changes in cash-flow risks.

According to the classical tradeoff theory, the firm should respond to an increase
in cash-flow risk by relying less on debt financing. This is indeed how our financially constrained firm responds when the level of cash-flow risks is low (with $\sigma$ increasing from 4% to 10%). As Panel A shows, the firm progressively lowers the coupon $b$ from 0.093 to 0.08. In this region, the firm chooses debt primarily for the tax shield purpose as the precautionary demand for cash is low due to low cash-flow risks. As a result, there is excess cash that is paid out immediately after debt issuance ($W_0 - \bar{W} > 0$; see Panel B). As cash-flow risk rises, the firm reduces the amount of debt issued, which causes the initial cash holding $W_0$ to fall as well, until it is equal to the payout boundary $\bar{W}$ when the volatility rises to $\sigma = 10\%$.

When $\sigma$ increases further from 10% to 16%, the firm responds by issuing more debt: the coupon increases from 0.08 to 0.088. The reason behind this increased reliance on debt is that the financially constrained firm at that point prefers to raise its initial cash buffer $W_0$ and further delay its payout to hedge against the increased cash-flow risk. Since the firm chooses not to issue any outside equity in the meantime (see Panel C), issuing more term debt is the only way to boost the firm’s initial cash holding.

Next, when $\sigma$ further increases from 16% to 40%, the firm again responds by reducing the amount of debt issued. As we see in Panel C, in this region the firm starts to issue

Figure 3: Optimal long-term debt, payout, outside equity share as functions of cash-flow volatility $\sigma$. 
outside equity, with outside equity share $a$ rising from 0 to 0.6. This is because when cash-flow volatility reaches sufficiently high levels ($\sigma > 0.16$), debt financing becomes more costly than equity due to the toll of debt servicing costs on corporate liquidity. As a result, the firm substitutes away from debt and into equity financing instead.

6 Large Liquidity Shocks

In our baseline model, the firm is only exposed to small cash-flow shocks. In reality, firms are also facing the risks of large liquidity shocks, as in Holmstrom and Tirole (1997), and Biais, Mariotti, Rochet, and Villeneuve (2010). Examples of such shocks include a sudden failure of productive asset or unexpected large operating losses. They will have qualitatively different effects on the firm’s liquidity management and capital structure decisions from the small diffusive shocks (as modeled via the Brownian shocks in equation (1)) we have considered so far. Our continuous-time framework is well-suited to examine the distinctive effects of small and large liquidity shocks.\footnote{Hugonnier and Morellec (2014) study liquidity management for financial institutions in the presence of large shocks.}

Specifically, we model the large liquidity shock in the form of a lumpy capital expenditure $I$, which is timed according to a Poisson arrival process with mean arrival rate $\zeta$. For simplicity, we fix $I$ to be a constant. The firm may want to increase its liquidity reserve in anticipation of such shocks.\footnote{One alternative interpretation for these liquidity shocks is the stochastic depreciation of physical capital. In practice, firms are allowed to set aside tax-deductible depreciation allowances, which can be captured by letting the firm subtract the expected depreciated amount of capital, $\zeta I$, from its taxable earnings in every period.} Following the realization of a large liquidity shock the firm can continue to operate as long as it has sufficient liquidity to cover the capital expenditure $I$, i.e., as long as $W \geq W^* \equiv I - C$. If the firm has insufficient liquidity, it is forced to liquidate. Without loss of generality, we focus on the case where $W^* > 0$ (or equivalently $I > C$). In addition, we restrict our attention to the case where the firm’s liquidation value is sufficiently low relative to the debt issued such that the equity value will be 0 at liquidation, but it is sufficiently high relative to the credit-line limit $C$ and the size of the large liquidity shock to ensure the credit line remain risk-free ($L - C - I > 0$). The
we calibrate the model by setting the size of the shock to $I = 0.25$ and the shock intensity to $\zeta = 0.15$. These parameter choices imply that the large shocks happen on average once per 6.7 years, and the size of these shocks is 2.5 times the annualized volatility of cash flows $\sigma$ in the baseline model. In order to make the model with large liquidity shocks more comparable with the baseline model, we compensate for the impact of the large liquidity shocks on firm value by increasing the mean productivity $\mu$ in the baseline model by the expected annual loss $\zeta I$. The remaining parameters take the baseline parameter values in Table 1.

The model solution is plotted in Figure 4. The main qualitative change induced by the large liquidity shock is the appearance of local convexity of the equity value $E(W)$ around $W^* = 0.15$. As Panel B shows, $E'(W)$ is increasing to the left and decreasing to the right of $W^*$. This local convexity of $E(W)$ is due to the fact that the firm will be forced to liquidate and the equity value will drop to zero whenever the large liquidity shock hits on the left side of $W^*$. Intuitively, it reflects the well-known benefit of gambling for resurrection for equity holders when $W$ is close to $W^*$. If $W$ is sufficiently far away
from $W^*$, the precautionary motive for holding cash dominates again, either because the firm has no hope of surviving a large liquidity shock or because it is not concerned about surviving the shock, which makes $E(W)$ concave as in the baseline model.

Another model that generates a similar local convexity pattern is the model of a financially constrained firm with lumpy investment of Hugonnier, Malamud, and Morellec (2014). Such a non-concave region, of course, has important implications for the firm’s optimal hedging policy. It is beyond the scope of this paper to provide a detailed analysis of optimal hedging as in Bolton, Chen, and Wang (2011, 2013). However, based on the insights of that analysis we can infer that the firm will optimally switch from a hedging policy seeking to reduce cash-flow volatility when $W$ is close to $-C$ and when $W$ is sufficiently larger than $W^*$, to a policy seeking to load up on cash-flow volatility when $W$ is in a neighborhood of $W^*$.

Besides the convexity in equity value, the firm responds to the threat of large liquidity shocks by: i) raising more initial funds ($W_0$ increases from 0.303 to 0.462); ii) holding more cash ($\bar{W}$ increases from 0.362 to 0.711), and iii) relying more on outside equity ($\alpha$ increases from 0 to 0.129). The firm’s market leverage and book leverage (as measured by the interest coverage) actually increase in the presence of large liquidity shocks ($L_0$ increases from 0.635 to 0.647 and interest coverage drops from 1.497 to 1.377).

7 Conclusion

Although the tax-advantage of debt has long been recognized as an important consideration for corporate financial policy, the tradeoff theory has had an uncertain standing, with many empirical studies concluding that it is flat-out rejected by the data. We have shown that one reason why the tradeoff theory performs poorly empirically is that it only applies to financially unconstrained firms. In the presence of external financing costs, firms’ financial policy is more complex and involves both an asset/liability management dimension and a liquidity management dimension. Thus, when there is a change in tax policy, for example, financially constrained firms generally have two margins along which they can respond:
they can either adjust their debt policy or their cash policy (or both).

As we have shown, the cash management dimension of corporate financial policy radically modifies the classical tradeoff theory. So much so that the theory for unconstrained firms provides very misleading predictions for corporate financial policies of constrained firms. Our model quantifies an important new cost of debt financing for financially constrained firms, the *endogenous debt servicing cost*, which helps explain the “debt conservatism puzzle” highlighted in the empirical literature. Moreover, our model provides a novel characterization of the net tax benefit of debt for a financially constrained firm, which succinctly captures the impact of liquidity constraint on capital structure decisions through the marginal value of liquidity. In contrast, the classical Miller formula for the effective tax rate only holds in the special case when the firm is unconstrained. Finally, our analysis of the tax implications for corporate financial policies of financially constrained firms is also relevant to the fiscal policy debate on changing the corporate tax rate.

There are several possible directions to extend our baseline analysis. *(i)* We have considered the case in which the firm only faces i.i.d. cash-flow shocks. The addition of persistent shocks will allow us to jointly analyze liquidity risks and solvency risks for financially constrained firms. *(ii)* It will also be useful to study the cases where subsequent external financings are allowed after the initial round, which is less extreme than the liquidation assumption we have adopted in this paper and makes the capital structure decisions fully dynamic. *(iii)* We discussed in Section 4 how the conflicts of interest between debt- and equity-holders can arise due to their different exposures to liquidity risks. Our model can be used to examine how debt covenants can be used to address this type of agency problems. *(iv)* it will be interesting to study the interactions of financing constraints and on-going corporate investment decisions in the presence of term debt.
Appendix

A Model Solution

This section provides more technical details for the model solution in Section 3.

There are two sub-regions in the interior region, the cash-hoarding region and the credit region. In the interior credit region, $-C \leq W \leq 0$, the firm’s after-tax credit evolution equation is given by

$$dW_t = (1 - \tau_c) (\mu + (r + \delta)W - \nu(C)(W_t + C) - b) \, dt + (1 - \tau_c) \sigma dZ_t. \quad (A1)$$

In the interior cash-hoarding region, $0 < W \leq \overline{W}$, the firm’s after-tax cash accumulation is given by

$$dW_t = (1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) \, dt + (1 - \tau_c) \sigma dZ_t. \quad (A2)$$

In the interior cash hoarding region $0 \leq W \leq \overline{W}$, equity value $E(W)$ satisfies the following ODE:

$$(1 - \tau_i) rE(W) = (1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) E'(W) + \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W). \quad (A3)$$

Note that we discount the after-tax cash flow using the after-tax discount rates $(1 - \tau_i) r$, as the alternative of investing in the firm’s equity is to invest in the risk-free asset earning an after-tax rate of return $(1 - \tau_i)r$.

Next, we turn to the interior credit region, $-C \leq W \leq 0$. Using a similar argument as the one for the cash hoarding region, $E(W)$ satisfies the following ODE:

$$(1 - \tau_i) rE(W) = (1 - \tau_c) (\mu + (r + \delta)W - \nu(C)(C + W) - b) E'(W) + \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W). \quad (A4)$$

Note that the firm pays the spread $\delta$ over the risk-free rate $r$ on the amount $|W|$ that it
draws down from its LOC.\textsuperscript{15}

Taking the firm’s payout policy $\overline{W}$ as given, investors price debt accordingly. In the cash hoarding region, $D(W)$ satisfies the following ODE:

$$
(1-\tau_i)rD(W) = (1 - \tau_i) b + (1-\tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) D'(W) + \frac{1}{2}\sigma^2(1-\tau_c)^2D''(W).
$$

(A5)

In the credit region, $-C < W < 0$, the ODE for debt pricing $D(W)$ is

$$
(1-\tau_i)rD(W) = (1 - \tau_i) b + (1-\tau_c) (\mu + (r + \delta)W - \nu(C+W) - b) D'(W) + \frac{1}{2}\sigma^2(1-\tau_c)^2D''(W).
$$

(A6)

At time 0, the entrepreneur chooses the fraction of outside equity $a$, the coupon on the perpetual risky debt $b$, and the credit line limit $C$ (with implied $W_0$) to solve the following problem:

$$
\max_{a,b,C} (1-a) E(W_0; b, C),
$$

(A7)

where

$$
W_0 = W_{0-} + F + P - (\gamma_E F + \gamma_D P + \Phi) - K,
$$

(A8)

$$
F = aE(W_0; b, C), \text{ and}
$$

(A9)

$$
P = D(W_0; b, C).
$$

(A10)

Without loss of generality we set $W_{0-} = 0$. The optimal amount of cash $W_0$ the firm starts out with (after the time-0 financing arrangement) is then given by the solution to the following equation, which defines a fixed point for $W_0$:

$$
(1 - \gamma_E) aE(W_0) + (1 - \gamma_D) D(W_0) = W_0 + K + \Phi.
$$

(A11)

\textbf{Solution procedure.} We now briefly sketch out our approach to solving numerically for the optimal capital structure at date 0. We focus our discussion on the most complex

\textsuperscript{15}Using a standard argument, one can show that $E(W)$ is continuously differentiable at $W = 0$. See Karatzas and Shreve (1991).
solution where the firm issues both debt and equity. The objective function in this case is given by (7). We begin by fixing a pair of \((b, C)\) and solving for \(E(W)\) and \(D(W)\) from the ODEs for \(E\) and \(D\). We then proceed to solve for the range of \(a\), as specified by \((a_{\text{min}}, a_{\text{max}})\), for which there is a solution \(W_0\) to the budget constraint (A8). Next, we solve for \(W_0\) from the fixed point problem (A8) for a given triplet \((b, C, a)\). There is either one or two fixed points, each representing an equilibrium. The intuition for the case of multiple equilibria is that outside investors can give the firm high or low valuation depending on the initial cash holding \(W_0\) being high or low, which in turn result in the actual \(W_0\) being high or low. Finally, we find \((b^*, C^*, a^*)\) that maximizes \((1 - a)E(W_0; b, C)\).

B Solution for the Model with Large Liquidity Shocks

In the presence of large liquidity shocks, equity value \(E(W)\) satisfies the following ODE in the region \(W^* \leq W \leq W\):

\[
(1 - \tau_i) rE(W) = (1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) E'(W) + \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W) + \zeta (E(W - I) - E(W)).
\]  

(A12)

The ODE (A12) differs from (A3) in the baseline model in the last term \(\zeta (E(W - I) - E(W))\), which reflects the impact of a large liquidity shock on equity value.

Next, in the region \(0 \leq W < W^*\), the ODE for \(E(W)\) becomes:

\[
(1 - \tau_i) rE(W) = (1 - \tau_c) (\mu + (r - \lambda)W - \nu(C)C - b) E'(W) + \frac{1}{2} \sigma^2 (1 - \tau_c)^2 E''(W) + \zeta (0 - E(W)).
\]  

(A13)

The ODE (A13) differs from (A12) in that the arrival of a large liquidity shock in this region will force immediate liquidation and drive the equity value to zero.
Finally, if $W < 0$, the ODE for $E(W)$ is given by:

$$(1 - \tau_i) r E(W) = (1 - \tau_e) (\mu + (r + \delta)W - \nu(C) (W + C) - b) E'(W) \quad \text{(A14)}$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_e)^2 E''(W) + \zeta (0 - E(W)).$$

The boundary conditions for $E(W)$ at the endogenous payout boundary $W = \overline{W}$ and the liquidation boundary $W = -C$ are identical to those in the baseline model, as given by equations (14) and (17). The fact that $E(W)$ is continuously differentiable at $W = 0$ and $W = W^*$ gives us four additional conditions. Finally, the same super-contact condition (15) determines the optimal payout boundary $\overline{W}$.

The value of debt $D(W)$ satisfies the following ODE in the region $W^* \leq W \leq \overline{W}$:

$$(1 - \tau_i) r D(W) = (1 - \tau_i) b + (1 - \tau_e) (\mu + (r - \lambda)W - \nu(C)C - b) D'(W)$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_e)^2 D''(W) + \zeta (D(W) - D(W)). \quad \text{(A15)}$$

In the region $0 \leq W < W^*$, the ODE for $D(W)$ is:

$$(1 - \tau_i) r D(W) = (1 - \tau_i) b + (1 - \tau_e) (\mu + (r - \lambda)W - \nu(C)C - b) D'(W)$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_e)^2 D''(W) + \zeta (L - I + W - D(W)). \quad \text{(A16)}$$

The last term follows from the fact that when a large liquidity shock arrives for $W < W^*$, the value of debt is equal to the liquidation value $L$ net of the loss $I$ plus remaining cash (or minus the credit line $-W$ if $W < 0$). Finally, when $W < 0$, the ODE for $D(W)$ is:

$$(1 - \tau_i) r D(W) = (1 - \tau_i) b + (1 - \tau_e) (\mu + (r + \delta)W - \nu(C)(W + C) - b) D'(W)$$

$$+ \frac{1}{2} \sigma^2 (1 - \tau_e)^2 D''(W) + \zeta (L - I + W - D(W)). \quad \text{(A17)}$$

The boundary conditions for $D(W)$ at the liquidation boundary $W = -C$ and the payout boundary $W = \overline{W}$ are the same (18) and (19). The fact that $D(W)$ is continuously differentiable at $W = 0$ and $W = W^*$ also gives us four additional conditions.

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References


